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Thermal and Dynamical Stability of Neutron Stars with Frictionally Coupled Cores and Crusts

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ABSTRACT

We study the thermal and dynamical evolution of the neutron star with frictional heat generation in its interior. We employ the simple two component model star with the superfluid core and the crust and consider the various core-crust coupling mechanisms proposed so far. The evolutionary stage of the neutron star is classified in terms of thermal and dynamical equilibria or disequilibria and the stability of each characteristic equilibrium state is examined by the perturbation analysis. We find that when the core-crust coupling has a strong dependence on temperature such as exponential dependence, the equilibrium state in which the cooling and heating rates balance is thermally unstable below a critical temperature. This critical temperature is determined by the condition that in the unstable regime the thermal time scale is shorter than the dynamical time scale. When coupling does not or weakly depends on temperature, however, there exist no unstable mode in any characteristic equilibrium state. Combining our results with the most recent study on the core-crust coupling by strongly magnetized superfluid vortices indicates that the simple two component model star is thermally and dynamically stable through the whole course of its evolution.

I. INTRODUCTION

The neutron star is an intriguing object which has been investigated intensively through pulsars and X-ray stars. The knowledge on its interior has been obtained mainly from the study of pulse frequency variations in pulsing sources (Lamb 1985 and references therein) and the study of the thermal history (see Tsuruta 1986 for a review). The observation of the glitch (sudden jump in pulse frequency) (Radhakrishnan and Manchester 1969; Reichley and Downs 1969) has led Baym et al. (1969) to the suggestion that the neutron star contains the superfluid in its interior. They proposed the two component model in which the neutron star consists of the superfluid core and the crust. They interpreted the post-glitch relaxation as resulting from the coupling of core to crust and explained observations qualitatively.

The relaxation time of the post-glitch behavior is considered to reflect the interaction of the charged component to the neutron. Feibelman (1971) considered the electron scattering from the vortex excitation and calculated the velocity relaxation time which covers the observed values (~ weeks to months). Harding, Guyer and Greenstein (1978) showed that the scattering of normal neutron within vortex lines from phonons or impurities in the crust yields the much shorter relaxation time than the electron-vortex excitation scattering. Sauls, Stein and Serene (1982) argued that $^3P_2$ vortices give the different coupling from $^1S_0$ superfluid because $^3P_2$ vortex has a weak spontaneous magnetization in the vortex core region. The relaxation time by this process is about a year. Recently, Alpar et al. (1984a) have shown that the vortex line in the core is strongly magnetized due to the induced proton current and consequently the neutron superfluid in the core couples to charged particles and to the crust in a very short time, ~ 400 P(s), where P is the rotation period of the star. Hence, they concluded that the core superfluid cannot be responsible for the observed post-glitch relaxation. Instead, Alpar et al. (1984b) proposed that the glitch is caused by the sudden unpinning of the superfluid vortex in the inner crust and the post-glitch behavior is determined by the creep motion of the pinned vortex. This model gave a good fit to observations.

The crust-core coupling which determines the dynamical response of the star accompanies the frictional heat generation, which in turn influences the coupling itself. Hence, it is vital to consider the thermal and dynamical behaviors together in order to understand the evolution of the neutron star correctly. First, Greenstein (1975 and references therein for his older works and see also Harding, Guyer and Greenstein 1978) paid attention to the internal energy dissipation and suggested that the frictional heating may have a significant effect on the later part of the thermal history of the star. Greenstein (1979) examined also the stability of this energy dissipation using the electron-vortex excitation scattering and showed that the frictional heat generation undergoes a thermal runaway. Recently, Shibazaki and Lamb (1986) calculated the cooling curve of the neutron star taking into account the internal energy dissipation due to the vortex creep. They found that the internal heating produces the considerably slower photon cooling phase which completely change the conventional picture of steep cooling of the old neutron star.

In this paper we study the thermal and dynamical evolution of the neutron star, especially paying attention to the thermal and dynamical stability of the characteristic stages which the star might undergo in its evolutionary course. We consider the simple two component model star with frictionally coupled core and crust. The thermal and dynamical stability of the vortex creep motion will be described in the next paper.

In Section II we present the basic equations to describe the thermal and dynamical evolution of the star. The characteristic stages of evolution are also mentioned. In Section III we apply the infinitesimal perturbation analysis to one characteristic stage and derive the criterion for the instability. In Section IV we study the physical meaning of the instability found in Section III. In Section V the stability of other characteristic stages is examined. In Section VI we discuss the possibility of the instability to occur.

II. FRICTIONAL COUPLING MODEL

We use the simple two-component model (Baym et al. 1969) in which the neutron star consists of the crust and the superfluid core. We assume that the rotation of each component is uniform and both components are coupled through the friction. We assume, furthermore, that the neutron star is isothermal. Here we consider first the basic equations to describe the thermal and rotational evolution of this model neutron star and then the equilibrium states which are conceivable in its evolutionary course.

The rotational motion of the crust is governed by the external torque N_{ext} and the internal friction torque:

$$I_c \dot{\Omega}_c = N_{ext} - I_s \dot{\Omega}_s \quad , \tag{1}$$

where Ω_c and Ω_s are the angular velocities of crust and core, respectively and I_c and I_s are the moments of inertia of crust and core, respectively.

Using the coupling time τ_c between the core and the crust, the equation of motion of the core is written as

$$\dot{\Omega}_s = -\frac{I_c}{I} \frac{\Omega_s - \Omega_c}{\tau_c} \quad , \tag{2}$$

where I is the total moment of inertia given by $I = I_s + I_c$. The several coupling mechanisms have been discussed in the past. Those results are expressed by the general form as given by

$$\tau_c = \frac{A}{T^m \Omega_s} \exp(T_a/T) \tag{3}$$

where T is the temperature, T_a the activation temperature and A the coupling constant. The coupling mechanisms considered so far are summarized in Table1.

The thermal history of the neutron star is described by

$$C_V\dot{T} = H - \Lambda \quad , \tag{4}$$

where C_V is the heat capacity, H the heating rate and Λ the cooling rate. Throughout this paper we employ the heat capacity for the degenerate matter, which is proportional to the temperature:

$$C_V = aT \quad , \tag{5}$$

where a is the constant. As a heat source we take into account only the friction between two components:

$$H = -I_s(\Omega_s - \Omega_c)\dot{\Omega}_s \quad . \tag{6}$$

The neutron star cools by the neutrino emission when it is hot. At low temperature the radiative cooling becomes dominant. The cooling rate can be expressed in a general form as a function of temperature:

$$\Lambda = BT^n \quad , \tag{7}$$

where n varies in the range 2 to 8 and B is the constant.

There are two characteristic equilibria which the neutron star might undergo in the course of its evolution, the dynamical equilibrium and the thermal one. In dynamical equilibrium the crust and the core decelerate at the same rate. Hence, dynamical equilibrium is specified by

$$\dot{\Omega}_{co} = \frac{N_{exto}}{I} = \dot{\Omega}_{so} = -\frac{I_c}{I}\frac{\Omega_{so} - \Omega_{co}}{\tau_{co}} \quad . \tag{8}$$

Hereafter the suffix o denotes the equilibrium state. It should be noted that in Eq. (8) the coupling time τ_{co} is assumed to be much shorter than the spin-down time τ_{so} implicitly:

$$\tau_{co} << \tau_{so} \quad , \tag{9}$$

where the spin-down time is defined by

$$\tau_{so} = \Omega_{co}/(-\dot{\Omega}_{co}) \quad . \tag{10}$$

From Eqs. (8) and (10) the angular velocity lag between core and crust in dynamical equilibrium is expressed as

$$\omega_o \equiv \Omega_{so} - \Omega_{co} = \frac{I}{I_c}\frac{\tau_{co}}{\tau_{so}}\Omega_{co} \quad . \tag{11}$$

Equation (11) combined with Eq. (9) indicates that the equilibrium lag is very small as compared with Ω_{co} or Ω_{so} in the standard model of a neutron star with $I_c/I = 1$–10^{-2} :

$$\omega_o << \Omega_{co} \sim \Omega_{so} \quad . \tag{12}$$

In thermal equilibrium the cooling rate balances with the heating rate:

$$-I_s(\Omega_{so}-\Omega_{co})\,\dot{\Omega}_{so} = BT_o^n \quad . \tag{13}$$

The thermal equilibrium is possible when the cooling (or heating) time is much shorter than the spin-down time:

$$\tau_{cool} << \tau_{so} \quad , \tag{14}$$

where τ_{cool} is the cooling (or heating) time defined by

$$\tau_{cool} = C_V T_o / BT_o^n = [C_V T_o / I_s(\Omega_{so}-\Omega_{co})(-\dot{\Omega}_{so})] \quad . \tag{15}$$

The evolutionary stages of neutron star are characterized in terms of thermal and dynamical equilibria or disequilibria as shown in Fig. 1. If the characteristic stages in Fig. 1 were all realized in the course of the evolution, then the simple scenario of the evolution would be the one indicated by the dashed line. The neutron star is born in thermal and dynamical disequilibrium. Since the coupling time is very short at high temperature, the neutron star settles into the dynamical equilibrium immediately. The cooling due to the neutrino emission is dominant over the frictional heating at high temperature. Hence, the neutron star is not in thermal equilibrium. The neutron star stays at the stage B as long as the cooling dominates the heating. As the temperature decreases and the frictional heating becomes important, the neutron star gets into the stage C, where it is in thermal and dynamical equilibrium. As the temperature falls further, the coupling time becomes comparable to the spin-down time:

$$\tau_s \equiv \Omega_c/(-\dot{\Omega}_c) \sim \tau_c \quad . \tag{16}$$

At this point the crust decouples from the core (Greenstein 1975) and the neutron star enters into the stage D. Combining Eq. (16) with Eqs. (3), (8) and (13) yields the critical temperature below which the decoupling occurs. This critical temperature T_d is illustrated in Fig. 2 as a function of A and T_a for the case of electron-vortex excitation scattering. It is seen that the critical temperature for decoupling T_d depends strongly on the activation temperature. The physical parameters at the decoupling time are shown in Table 2 for the case of $A = 5\times10^{12}$ rad K and $T_a = 10^8$ K.

Depending on the initial condition, the neutron star model and the coupling mechanism, however, the simple evolutionary track mentioned above will be modified. Furthermore, it should be noted that the evolutionary path in Fig. 1 is also influenced by the stability or instability of each stage. If a certain stage is unstable, that stage may not persist and the subsequent stages may never be reached. In order to understand the thermal and rotational evolution of neutron star, hence, it is very important to see the stability or instability of the characteristic stages shown in Fig. 1. In the next sections we examine the stability of these stages.

III. STABILITY OF STAGE C TO INFINITESIMAL PERTURBATION

We begin the stability analysis with the stage C in which the thermal and dynamical equilibrium is attained since it is the most general case and the part of its analysis is also applied to the stages B and D.

(1) <u>Infinitesimal perturbation</u>

In order to know the stability or instability we introduce the infinitesimal perturbation into the stationary state and see whether or not it grows with time.

$$\Omega_c = \Omega_{co} + \delta\Omega_c, \;\; \Omega_s = \Omega_{so} + \delta\Omega_s, \;\; \text{and } T = T_o + \delta T, \tag{17}$$

where $\delta\Omega_c$, $\delta\Omega_s$ and δT are the infinitesimal perturbations of crust angular velocity, core angular velocity and temperature, respectively. From Eqs. (1), (2) and (4) together with Eqs. (3), (5)-(8), (13) and (17), we obtain the linearized equations:

$$\frac{I_c}{I}\frac{\delta\dot{\Omega}_c}{\dot{\Omega}_{co}} = \left(\frac{\partial \ln |N_{ext}|}{\partial \ln\Omega_c}\right)_o \frac{\delta\Omega_c}{\Omega_{co}} - \frac{I_s}{I}\frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} \tag{18}$$

$$\frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} = -\frac{\Omega_{co}\,\delta\Omega_c}{\omega_o\Omega_{co}} + \left(1+\frac{\Omega_{so}}{\omega_o}\right)\frac{\delta\Omega_s}{\Omega_{so}} + \left(m + \frac{T_a}{T_o}\right)\frac{\delta T}{T_o} \tag{19}$$

$$\frac{\delta\dot{T}}{\dot{T}_{cool}} = -\frac{\Omega_{co}}{\omega_o}\frac{\delta\Omega_c}{\Omega_{co}} + \frac{\Omega_{so}}{\omega_o}\frac{\delta\dot{\Omega}_s}{\Omega_{so}} + \frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} - n\frac{\delta T}{T_o} \quad , \tag{20}$$

where

$$\dot{T}_{cool} = T_o/\tau_{cool} \quad . \tag{21}$$

It is assumed in Eq. (18) that the external torque is a function of Ω_c. We consider the solution such as

$$\begin{aligned} \delta\Omega_c/\Omega_{co} &= \varepsilon_1 e^{-i\nu t} \\ \delta\Omega_s/\Omega_{so} &= \varepsilon_2 e^{-i\nu t} \\ \delta T/T_o &= \varepsilon_3 e^{-i\nu t} \quad , \end{aligned} \tag{22}$$

where ε_1, ε_2, and ε_3 are infinitesimal constants. In this paper we consider the solution which has the characteristic time much shorter than the spin-downtime:

$$1/|\nu| << \tau_{so} \quad . \tag{23}$$

When we take the time derivative of perturbations $\delta\Omega_c$, $\delta\Omega_c$ and δT in Eq. (22), therefore, Ω_{co}, Ω_{so} and T_o are kept constant.

In the following of this paper we ignore the effect of external torque. Because we found that the effect of the magnetic braking does not change the essential results on the instability and it makes the discussion simple to neglect the external torque. Now the neutron star is a closed system and hence the angular momentum is conserved through the perturbation:

$$I_c\delta\Omega_c + I_s\delta\Omega_s = 0 \quad . \tag{24}$$

We consider here the case where

$$I_c << I_s \tag{25}$$

(We find the same results also for the case $I_c >> I_s$). Equations (24) and (25) give

$$|\delta\Omega_s| << |\delta\Omega_c| \quad . \tag{26}$$

Under the condition of Eqs. (12) and (26) $\delta\Omega_s$ terms in Eqs. (19) and (20) can be neglected as compared with $\delta\Omega_c$ terms. Inserting Eq. (22) into Eqs. (18)-(20) under above assumptions (Eqs. (12), (23), (25) and (26)), we derive

$$\nu^2 + (\tau_{co}^{-1} - \xi\,\tau_{cool}^{-1})\, i\nu - \zeta\tau_{co}^{-1}\,\tau_{cool}^{-1} = 0 \quad , \tag{27}$$

where ξ and ζ are defined by

$$\xi = \frac{T_a}{T_o} + m - n \tag{28}$$

and

$$\zeta = \frac{T_a}{T_o} + m + n \quad , \tag{29}$$

respectively.

(2) Solutions

The property of the solution to Eq. (27) can be understood from examining the extreme cases.

(i) $\xi > 0$ and $\tau_{co} >> \tau_{cool} / \xi$

The solutions to Eq. (27) in this case are calculated as

$$\nu_1 \sim (\tau_{cool}/\xi)^{-1}\, i \tag{30}$$

and

$$\nu_2 \sim (\xi\,\tau_{co}/\zeta)^{-1}\, i \tag{31}$$

Since the imaginary part of ν is positive in both solutions, the perturbation grows with time and hence the neutron star is unstable. It is noted that the growth times ($\sim 1/|\nu|$) of the perturbation in solutions ν_1 and ν_2 are related to the cooling and coupling times, respectively. This evidence implies that the unstable modes ν_1 and ν_2 may be thermal and dynamical instabilities, respectively.

(ii) $\xi > 0$ and $\tau_{co} << \tau_{cool} / \xi$

In this case Eq. (27) yields two solutions given by

$$\nu_1 \sim -(\tau_{cool}/\zeta)^{-1}\, i \tag{32}$$

and

$$\nu_2 \sim -\tau_{co}^{-1}\, i \tag{33}$$

The negative imaginary part in both solutions shows that the perturbation damps with time and hence the neutron star is stable in this case.

(iii) $\xi < 0$

It is easily seen that solutions to Eq. (27) have all negative imaginary parts in this case and hence the star is stable.

It is to be noted that the coupling process with no temperature dependence in Table 1 corresponds to this case since setting $T_a = 0$ and $m = 0$ yields $\xi = -n < 0$. Hence, if the coupling between crust and core is determined by the electron scattering from magnetized vortex, the neutron star in stage C is stable.

(3) Instability criterion

It is known from the above analysis that the neutron star is unstable when the following conditions are satisfied:

$$\xi \equiv \frac{T_a}{T_o} + m - n > 0 \tag{34}$$

and

$$\tau_{to} \equiv \tau_{cool}/\xi < \tau_{co} \quad . \tag{35}$$

Equation (34) is the condition which Greenstein (1979) presented for the occurrence of thermal instability. Our study, however, shows that Eq. (34) is necessary, but not sufficient. In addition to Eq. (34), Eq. (35) is also required for the instability. In the case of the coupling process with exponential dependence on temperature, in fact, the instability is determined solely by Eq. (35) since in the situation of our interest $T_o \ll T_a$ and hence the inequality in Eq. (34) is always satisfied. Now we can conclude that the neutron star is unstable when the thermal time (τ_{to}) is shorter than the dynamical time (τ_{co}).

As seen from Eq. (3), the coupling time (dynamical time) becomes larger rapidly as the temperature decreases in the case $T_a \neq 0$. Whereas, the thermal time τ_{to} increases relatively slowly as the neutron star cools. Hence, there is a critical temperature T_c below which the neutron star becomes unstable. This critical temperature is depicted in Fig. 3 as a function of the coupling constant A and the activation temperature T_a for the case of electron-vortex excitation scattering. It is noted that the critical temperature depends strongly on the activation temperature, but very weakly on the coupling constant. The physical parameters at this critical moment for the case of $A = 5 \times 10^{12}$ rad K and $T_a = 10^8$ K are written in Table 2.

IV. PHYSICAL INTERPRETATION OF THE UNSTABLE MODE

In the previous section the two unstable modes have been found and suggested to be the thermal and dynamical instabilities in terms of their growth time. Here we study the physical meaning of these instabilities.

(1) Thermal response

First we pay attention to the thermal behavior of the neutron star when the temperature is perturbed. Under the condition of Eq. (35) the change in dynamical quantities Ω_c and Ω_s in Eqs. (19) and (20) can be neglected because the dynamical time is much longer than the thermal time. Hence, the response to the temperature perturbation is determined by

$$\frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} = \left(\frac{T_a}{T_o} + m\right)\frac{\delta T}{T_o} \tag{36}$$

and

$$\frac{\delta\dot{T}}{\dot{T}_{cool}} = \frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} - n\frac{\delta T}{T_o} \quad . \tag{37}$$

Equations (36) and (37) can be solved to give

$$\delta T(t) = \delta T(0)\exp(t/\tau_{to}) \quad . \tag{38}$$

The temperature perturbation grows with a time scale of τ_{to}. The unstable mode ν_1 in the previous section is truly identified as a thermal instability.

This thermal instability is understood physically as follows. As seen from Eqs. (3) and (2), the increase in temperature makes the coupling time shorter and the friction larger. The more rotational energy is dissipated into the heat, which results in further temperature increase. In this way the thermal runaway proceeds.

(2) Dynamical response

Next we consider the dynamical perturbation keeping the thermal equilibrium. Under the thermal equilibrium and the assumptions of Eqs. (12) and (26), Eqs. (18)-(20) reduce to

$$I_c \frac{\delta\dot{\Omega}_c}{\dot{\Omega}_{co}} = -I_s \frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} \tag{39}$$

$$\frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} = -\frac{\Omega_{co}}{\omega_o}\frac{\delta\Omega_c}{\Omega_{co}} + \left(\frac{T_a}{T_o} + m\right)\frac{\delta T}{T_o} \tag{40}$$

$$-\frac{\Omega_{co}}{\omega_o}\frac{\delta\Omega_c}{\Omega_{co}} + \frac{\delta\dot{\Omega}_s}{\dot{\Omega}_{so}} - n\,\frac{\delta T}{T_o} = 0 \quad . \tag{41}$$

The solution to Eqs. (39)-(41) is calculated as

$$\begin{aligned} \delta\Omega_c(t) &= \delta\Omega_c(0)\exp\left(t\Big/\frac{\xi}{\zeta}\tau_{co}\right) \\ \delta\Omega_s(t) &= -\frac{I_c}{I_s}\delta\Omega_c(0)\exp\left(t\Big/\frac{\xi}{\zeta}\tau_{co}\right) \\ \delta T(t) &= \delta T(0)\exp\left(t\Big/\frac{\xi}{\zeta}\tau_{co}\right) \quad , \end{aligned} \tag{42}$$

where $\delta T(0) = (2/\xi)(\delta\Omega_c(0)/\omega_o)T_o$. The perturbation grows with a time scale of the coupling time.

The physical cause of this instability is interpreted as follows. Consider the dynamical perturbation in which Ω_c increases and Ω_s decreases. The decrease in lag ω accompanies the dissipation of rotational energy into heat. Then the temperature increases. The higher temperature makes the coupling time shorter and hence the friction larger, which leads Ω_c and Ω_s to further increase and decrease, respectively.

The unstable mode v_2 found in the previous section is now identified as a dynamical instability. Even if this dynamical instability is triggered by some perturbation, it will be suppressed to proceed as compared with the thermal instability mentioned above, especially if triggered at $T_0 << T_c$. Because the thermal equilibrium assumed in the course of the dynamical perturbation is already unstable and this thermal instability grows faster than the dynamicl instability

V. STABILITY OF STAGES B AND D

Here we apply the perturbation analysis to the stages B and D and examine their stability.

(1) Stage B

It is assumed in the stage B that the neutron star is in dynamical equilibrium, but not in thermal equilibrium. This stage might be considered for the hot neutron star since at high temperature the coupling time is very short and the neutrino cooling dominates over the frictional heating. The stability of this state is judged from the response to the dynamical perturbation of the neutron star.

Since the cooling time τ_{cool} is much longer than the coupling time, the temperature is assumed to be constant through the dynamical perturbation. Neglecting the external torque and the temperature variation, Eqs. (18), (19) and (22) together with Eqs. (10) and (11) give

$$\nu^2 + i\,\alpha\tau_{co}^{-1}\nu = 0 \quad , \tag{44}$$

where

$$\alpha = 1 + \frac{I_c}{I}\frac{\omega_o}{\Omega_{so}} \quad . \tag{45}$$

Equation (12) indicates that $\alpha \sim 1$ in the situation as concerned. The solution which satisfies Eq. (23) is obtained as

$$\nu \sim -\left(\frac{\tau_{co}}{\alpha}\right)^{-1} i \quad . \tag{46}$$

The perturbation damps with the time scale of the coupling time and hence the neutron star in the stage B is stable.

(2) Stage D

The stage D is the thermal equilibrium state after the decoupling in which the coupling time is longer than the spin-down time. Dynamically the neutron star is not in equilibrium. After decoupling the crust is decelerated considerably and its angular velocity approaches to zero. Neglecting $\delta\Omega_c$ terms, Eqs. (19) and (20) together with Eq. (22) yield

$$(\tau_{so}\tau_{cool})\nu^2 + i\nu[(1+\frac{\Omega_{so}}{\omega_o})\tau_{cool} - \xi\tau_{so}] - (\zeta\frac{\Omega_{so}}{\omega_o} + n) = 0 \quad , \tag{47}$$

where τ_{so} redefined as $\tau_{so} \equiv \Omega_{so}/(-\dot{\Omega}_{so})$ is the spin-down time of the core. In the case of the coupling process with exponential dependence on temperature ($T_a \neq 0$), as seen from Fig. 2, the decoupling occurs at low temperature, $T_o << T_a$, which yields $\xi >> 1$. Since $\Omega_{co} \sim 0$ after decoupling, $\omega_o \sim \Omega_{so}$. Furthermore, if we take into account the condition for the thermal equilibrium given by Eq. (14), we can see that the first term in square bracket in Eq. (47) can be neglected. Then, the solution satisfying Eq.(23) is obtained as

$$\nu \sim \tau_{to}^{-1}\, i \quad . \tag{48}$$

The perturbation grows with the thermal time scale τ_{to} and hence the stage D is thermally unstable. The physical meaning of this instability is the same as that given in Section 4.

When $\xi < 0$ as in the case of the coupling process with no temperature dependence, it is easily known that Eq. (47) yields solutions with negative imaginary part. Since the perturbation diminishes with time, the star is thermally stable in this case.

VI. CONCLUDING REMARKS

We have examined the thermal and dynamical evolution of the neutron star with frictionally coupled core and crust examining the stability of the characteristic equilibrium states which are conceivable in its evolutionary course. The main results obtained are summarized as follows:

(1) The hot neutron star which is in dynamical equilibrium and in which the (neutrino) cooling rate dominates over the internal heating rate is stable irespective of the core-crust coupling mechanisms.

(2) If the core-crust coupling has a strong dependence on temperature such as exponential dependence, the thermal and dynamicl equilibrium state is stable above a critical temperature, but thermally unstable below a critical temperature. This critical temperature is

determined by the condition that in the unstable regime the thermal time scale is shorter than the dynamical time scale.

(3) The thermal equilibrium state after the decoupling of the crust from the core is also thermally unstable if the core-crust coupling has a strong temperature dependence.

(4) This thermal instability is derived from the sensitve dependence of the core-crust coupling on temperature. In the case of the coupling process with exponential dependence on temperature, the temperature increase produces the shorter coupling time and hence larger frictional heat generation which leads to still higher temperature and then results in a thermal runaway.

(5) If the core-crust coupling does not or weakly depend on temperature, the neutron star is thermally and dynamically stable through the whole course of its evolution.

Recently, Alpar et al. (1984a) found that $^3P_2$ vortex lines in the superfluid core of the star are strongly magnetized by the induced proton charge current and hence the electron scattering from vortices gives very short coupling time of core to crust. In fact, the coupling time presented by them suggests that this coupling process largely overwhelms the other processes considered so far and determines the coupling of core to crust for temperature below the superconducting transition temperature (~ 10^{10} K). It is to be noted, furthermore, that this coupling process does not depend on temperature. Hence, combining these facts with our results leads to the conclusion that the neutron star described by the simple two component model is stable through the course of its thermal and dynamical evolution.

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Table 1 Coupling Mechanism

Coupling Mechanism	Coupling Time
Electron-vortex excitation scattering[1]	$\frac{A}{T\Omega_s}\exp(T_a/T)$
2-Phonon N process[2]	$\frac{A}{T^{8.5}\Omega_s}\exp(T_a/T)$
2-Phonon U process[2]	$\frac{A}{T^{2.5}\Omega_s}\exp(T_a/T)$
Neutron-impurity scattering[2]	$\frac{AT}{\Omega_s}\exp(T_a/T)$
Spontaneous magnetization of $^3P_2$ vortices[3]	$\frac{A}{\Omega_s}$
Strong magnetization of $^3P_2$ vortices[4]	$\frac{A}{\Omega_s}$

[1]Feibelman (1971); [2] Harding et al. (1978);
[3]Sauls et al. (1982); [4] Alpar et al. (1984a)

Table 2 Physical parameters at the critical moments for the instability and the decoupling[1]

		Instability	Decoupling
Critical temperature	T_c, T_d(K)	6.4×10^6	4.0×10^6
Temperature increase[2]	ΔT(K)	2.2×10^5	2.5×10^8
Crust angular velocity	Ω_{co}(rad/s)	17	2.3
Core angular velocity	Ω_{so}(rad/s)	17	7.1
Angular velocity lag	ω_o(rad/s)	0.03	4.8
Angular deceleration	$\lvert\dot{\Omega}_{co}\rvert$ (rad/s^2)	5.7×10^{-14}	1.4×10^{-16}
Coupling time	τ_{co}(y)	8.7×10^3	5.2×10^8
Cooling time	τ_{cool}(y)	1.3×10^5	1.4×10^5
Spin-down time	τ_{so}(y)	9.5×10^6	5.2×10^8

1 The same model parameters as those in Fig. 2 are used. The coupling constant and the activation temperature are chosen as $A = 5\times10^{12}$ rad K and $T_a = 10^8$ K, respectively.

2 ΔT is the increase in temperature when the lag ω_o goes to zero suddenly and the rotational energy is dissipated into heat,

$$\int_{T_c}^{T_c+\Delta T} C_V dT = \frac{1}{2}\frac{I_c I_s}{I}\omega_o^2 \quad .$$

FIGURE CAPTIONS

Fig. 1 Characteristic evolutionary stage of the neutron star in terms of thermal and dynamical equilibria or disequilibria. The dashed line indicates the possible evolutionary track in the case where all characteristic stages are realized in the course of evolution.

Fig. 2 Temperature at the decoupling time as a function of the coupling constant and the activation temperature. The coupling model used is the Feibelman's case (1971). The stiff neutron star model (Pandharipande et al. 1976) is employed; $M = 1.4\ M_{\odot}$., $R = 1.58\times10^{6}$ cm, $I_s = 1.13\times10^{45}$ g cm^2 and $I_c = 1.05\times10^{45}$ g cm^2. The magnetic braking due to the magnetic dipole radiation is taken into account as the external torque. The magnetic dipole moment used is 10^{30} gauss cm^3. The photon cooling law and the heat capacity used are $\Lambda = 2.2\times10^{15}\ T^{2.2}$ ergs/s (Gudmundsson et al. 1983) and $C_V = 2\times10^{29}$ T erg/ K.

Fig. 3 Critical temperature for the instability as a function of the coupling constant and the activation temperature. The neutron star is unstable below the critical temperature. The model parameters used are the same as those in Fig. 2.

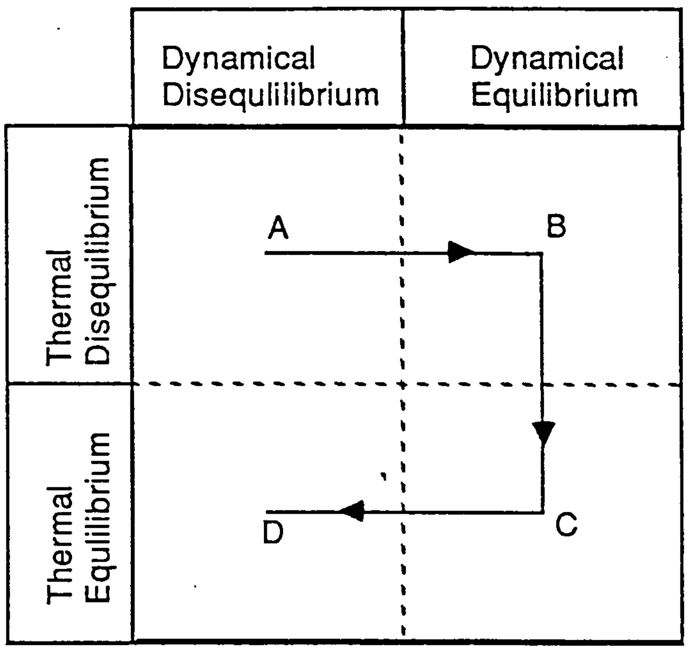
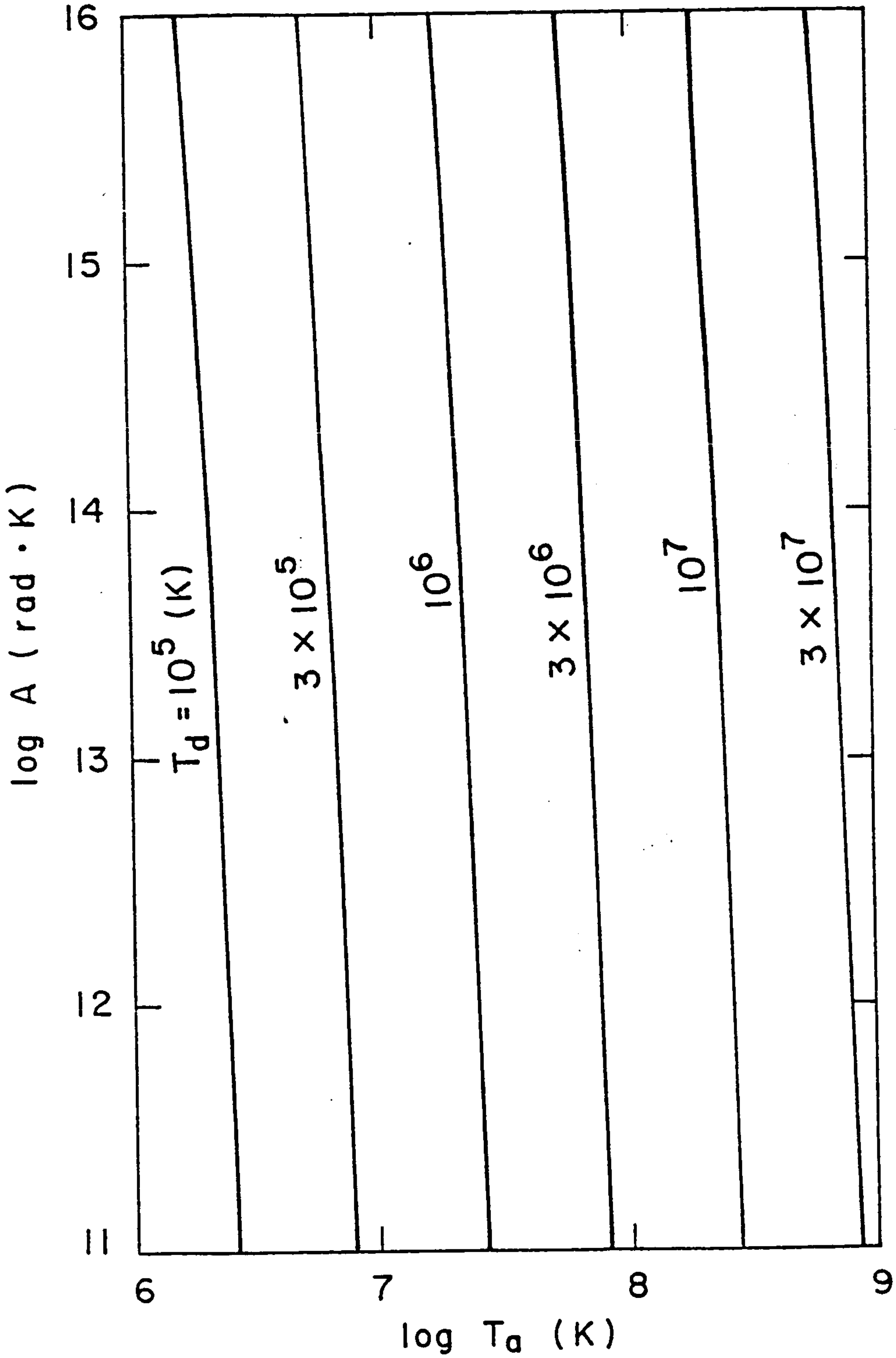
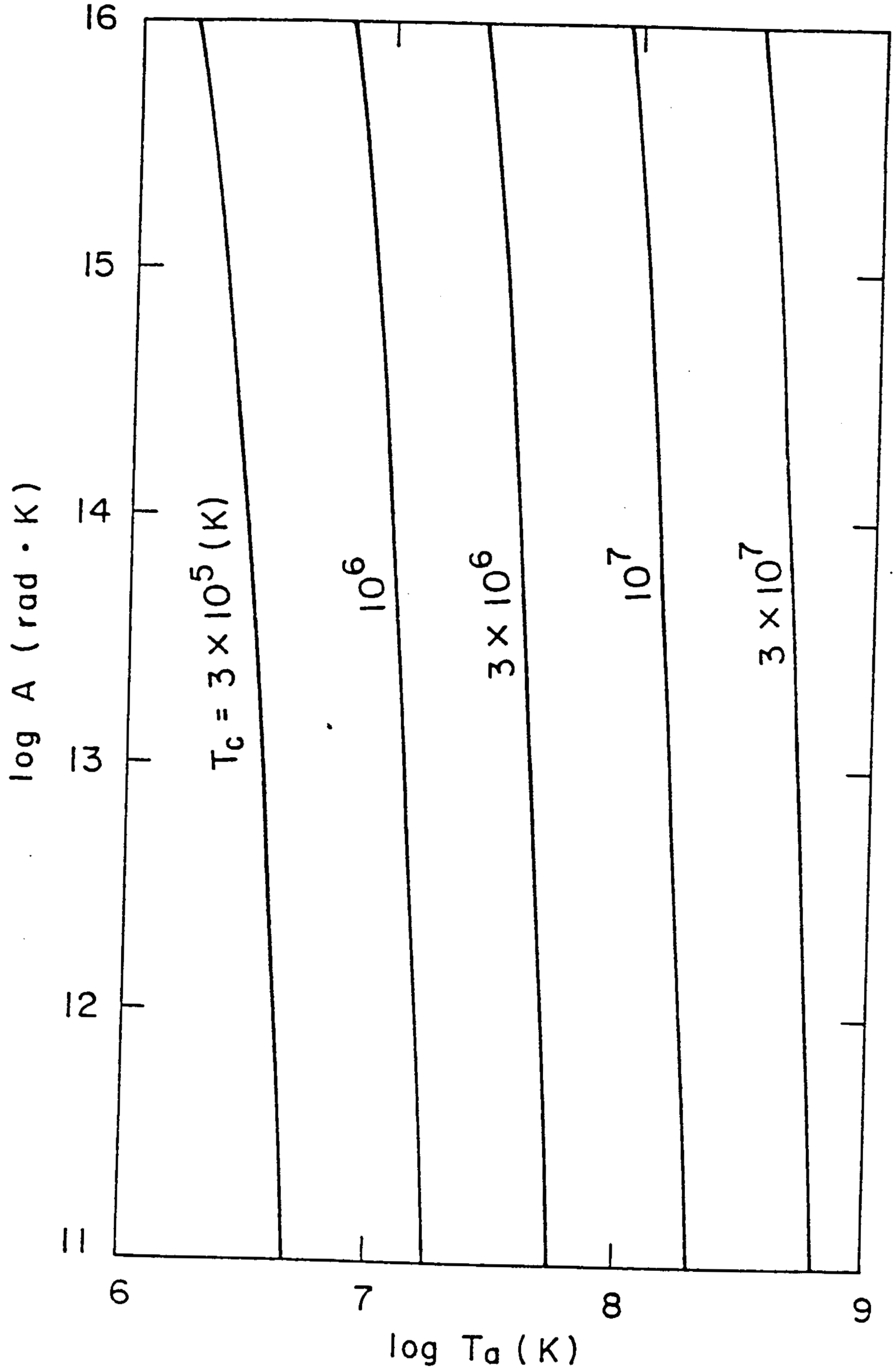

Dynamical
Disequlilibrium
Dynamical
Equilibrium
Thermal
Disequilibrium
Thermal
Equilibrium
A
B
C
D

Fig.1



log A (rad · K)
16
15
14
13
12
11
6
7
8
9
log Ta (K)
$T_d = 10^5$ (K)
3×10^5
10^6
3×10^6
10^7
3×10^7



log A (rad · K)
log Ta (K)
16
15
14
13
12
11
6
7
8
9
$T_c = 3 \times 10^5$ (K)
10^6
3×10^6
10^7
3×10^7